Thermodynamic optimization of convective heat transfer in a turbulent flow through a pipe with constant wall temperature

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Abstract
One of the main goals in different engineering fields, especially mechanical engineering, is to reduce the usage of energy. Heat exchangers are vastly being used in industry, so it of a great value to design heat exchangers at their optimum point in order to make use of energy efficiently. In the present work a thermo dynamical model of conventional cylindrical pipe, which is used in heat exchangers, is developed based on the second law of thermodynamics. Entropy generation, an important parameter of the heat pipe performance, is produced by the temperature difference between the hot and cold reservoirs, the frictional losses in the working fluid flows, and the vapor temperature/pressure drop along the heat pipe. Different parameters can be adjusted to minimize entropy generation in the system and that's what is done in the present work.
Keywords: second law analysis, Exergy, Entropy

Introduction
Saving of energy is a primary objective in all engineering fields, especially mechanical and energy engineering, and that makes Efficiency somehow the most important factor in designing a thermo dynamical system. The second-law analysis is the gateway for optimization in thermal equipments and systems, which makes good engineering sense to focus on the irreversibilities of fluid flow and heat transfer processes. Entropy, a main characteristic of every thermo dynamical system, is defined by the second law of thermodynamic. The total entropy of a system consists of two terms, a term which is produced by heat transfer and the entropy generation term which is produced due to the irreversibilities. Minimization of entropy generation in a thermo dynamical system provides efficient use of available energy. Fluid flow and heat transfer inside a circular duct for different boundary conditions are a fundamental area of research in engineering. Circular ducts appear in many engineering applications as a single unit or in combination, such as in heat exchangers used in power and process industries to transfer heat from one fluid stream to another. The working fluid inside the pipe undergoes a thermo dynamic cycle which generates entropy. Entropy generation or Exergy destruction due to heat transfer and fluid flow through a duct has been investigated by many researchers and non-dimensional entropy generation number is always employed in the irreversibility examination of convective heat transfer.

Bejan [1, 2] has done an invaluable job on founding the basic method of second law analysis in heat transfer and the development of the steps to find the total entropy generation in a heat transfer problem. Sahin [3, 4] has considered the effects of the change of viscosity in entropy generation for a heating process of a duct with constant heat flux. He has also calculated the required pumping power. The flow is considered to be a laminar viscous flow in his study. Entropy is generated due to several factors including friction and as heat transfer enhancement equipments usually increase friction factor, the effects of some heat transfer equipments on the total entropy generation are studied by Perez-Blanco [5] and the optimum points are introduced. Abolfazli and Alizadeh [6] have also studied the thermodynamic optimization of geometry in convective heat transfer by considering the flow to be laminar and the walls to be at constant temperature. In their study they have researched the effects of different parameter on the Entropy generation and pumping power and have tried to introduce a proper correlation for the optimum design of the tube.

In the present work the formulation and calculations of entropy generation through a pipe with constant wall temperature is done by considering the flow to be turbulent and the minimum point for entropy generation is introduced as the optimum point for the design of the pipe in different usages like conventional heat exchangers.

Methodology

Let us consider the constant cross sectional area duct shown in Fig.1. The surface temperature of the duct is kept constant at T_w. An incompressible viscous fluid with mass flow rate,$\dot{m}$, and inlet temperature, T_i,enters the duct of length L. Heat transfer to the bulk of fluid occurs with an average heat transfer coefficient $\bar{h}$ at the inner surface. We also assume all properties of the fluid to be independent of the variation of the fluid temperature. Within the range of velocity considered in this study (Table1) the fluid flow assumed tube fully developed turbulent flow. The rate of heat transfer to the fluid inside the control volume show in Fig.1 is:

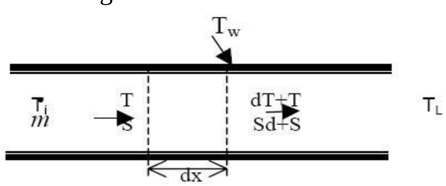

Fig.1. Constant cross sectional area duct

$$\delta\dot{Q} = \dot{m}C_p dT = \bar{h}\pi D(T_w - T)dx \qquad (1)$$

$$\dot{m} = \frac{\rho\bar{U}\pi D^2}{4} \qquad (2)$$

Substitute eq.2 in eq.1 and Integrate =>

$$T = T_w - (T_w - T_i)\exp(-\frac{4hx}{\rho\bar{U}DC_p}) \qquad (3)$$

Rate of entropy generation from the second law of Thermodynamic:

$$d\dot{S}_{gen} = \dot{m}ds - \frac{dQ}{T_w} \qquad (4)$$

And for an incompressible flow:

$$ds = \frac{C_p dT}{T} - \frac{dP}{\rho T} \qquad (5)$$

Now by substituting eq.5 and eq.1 into eq.4 we get :

$$d\dot{S}_{gen} = \dot{m}C_p(\frac{T_w - T}{T_w T}dT - \frac{dP}{\rho C_p T}) \qquad (6)$$

We need to substitute the equivalent pressure drop in the above eq. so that we could integrate the equation and obtain $\dot{S}_{gen}$:

$$dP = -\gamma\, dh_f \qquad (7)$$

$$dh_f = \frac{fU^2}{2gD}dx \qquad (8)$$

Where the friction factor for a fully developed turbulent flow in a flat tube is:

$$f = \frac{0.316}{R^{\frac{1}{4}}} \qquad (9)$$

$$R = \frac{\rho\bar{U}D}{\mu} \qquad (10)$$

So by substituting eq. 9 & 10 into 8 and then by putting eq.8 and eq. 3 into eq. 6 we get eq.11:

$$d\dot{S}_{gen} = \dot{m}C_p(\frac{4h(T_w - T_i)}{\rho\bar{U}DC_p} * \frac{\left(T_w - \frac{T_i}{T_w}\right) * \exp\left(-\frac{4hx}{\rho\bar{U}DC_p}\right) * \exp\left(-\frac{4hx}{p\bar{U}DC_p}\right)}{T_w - (T_w - T_i)\exp\left(-\frac{4hx}{\rho\bar{U}DC_p}\right)}dx + \left(\frac{0.316}{2C_p}\right)\left(\frac{\mu}{\rho}\right)^{\frac{1}{4}}\left(\frac{U^{\frac{7}{4}}}{D^{\frac{9}{4}}}\right) * \frac{dx}{T_w - (T_w - T_i)\exp\left(-\frac{4hx}{p\bar{U}DC_p}\right)})$$

(11)

By assuming:

$$\tau = \frac{T_w - T_i}{T_w}, \quad a = \frac{4h}{\rho\bar{U}DC_p} \quad , \quad B = \left(\frac{0.316}{2C_p}\right)\left(\frac{\mu}{\rho}\right)^{\frac{1}{4}}\left(\frac{U^{\frac{7}{4}}}{D^{\frac{9}{4}}}\right) \quad , \quad C = \frac{B}{T_w}$$

We can rewrite eq.11 this way:

$$d\dot{S}_{gen} = \dot{m}C_p((a\tau^2) * \frac{[\exp(-ax)]^2 dx}{1 - \tau\exp(-ax)} + C\frac{dx}{1 - \tau\exp(-ax)})$$

(12)

By integrating eq.12 with respect to variable x we get:

$$\dot{S}_{gen} = \dot{m}C_p(\tau\exp(-ax) + \ln(-1 + \tau\exp(-ax)) - \frac{C}{a}\ln(\exp(-ax)) + \frac{C}{a}\ln(-1 + \tau\exp(-ax)))$$

(13)

So the integration along the tube length of L is:

$$\dot{S}_{gen} = \dot{m}C_p(\tau\exp(-aL) + \ln(-1 + \tau\exp(-aL)) - \frac{C}{a} * \ln(\exp(-aL)) + \frac{C}{a}\ln(-1 + \tau\exp(-aL)) - \left[\tau + \left(1 + \frac{C}{a}\right)l(-1 + \tau)\right])$$

(14)

By defining:

$St = \frac{h}{\rho \bar{U} C_p} = \frac{Nu}{Re.pr}$, $a = \frac{4St}{D}$, $Sd = St.\frac{L}{D}$, $E = \frac{C}{\frac{4}{D}\frac{Nu}{Re.Pr}}$

Let's rewrite eq.14

$$\frac{\dot{S}_{gen}}{\dot{m}C_p} = \tau\exp\left(-4St\frac{L}{D}\right) + \ln\left(-1+\tau\exp\left(-4St\frac{L}{D}\right)\right) - \frac{C}{\frac{4}{D}\frac{Nu}{Re.Pr}}\ln\left(\exp\left(-4St\frac{L}{D}\right)\right) + \frac{C}{\frac{4}{D}\frac{Nu}{Re.Pr}}\ln\left(-1+\tau\exp\left(-4St\frac{L}{D}\right)\right) + [\tau + (1-\frac{C}{\frac{4}{D}\frac{Nu}{Re.Pr}})\ln(-1+\tau)]$$

(15)

Dimensionless entropy generation would be:

$$\psi = \frac{\dot{S}_{gen}}{\frac{\dot{Q}}{T_w - T_i}} \qquad (16)$$

Where $\dot{Q}$ is:

$$\dot{Q} = \dot{m}C_p(T_l - T_i) = \dot{m}C_p(T_w - T_i)[1 - \exp(-4St.\tfrac{L}{D})] \qquad (17)$$

$$\frac{\dot{Q}}{T_w - T_i} = \dot{m}C_p[1 - \exp(-4Sd)] \qquad (18)$$

So eq.16 would be:

$$\dot{\psi} = \frac{\tau\exp(-4Sd)}{[1-\exp(-4Sd)]} + \frac{\ln(-1+\tau\exp(-4Sd))}{[1-\exp(-4Sd)]} - E\frac{\ln(\exp(-4Sd))}{[1-\exp(-4Sd)]} + E\frac{\ln(-1+\tau\exp(-4Sd))}{[1-\exp(-4Sd)]} + \frac{[\tau+(1-E)\ln(-1+\tau)]}{[1-\exp(-4Sd)]} \qquad (19)$$

The total dimensionless entropy consists of two distinct parts so we can write:

$$\dot{\psi}_{total} = \dot{\psi}_{thermal} + \dot{\psi}_{frictional} \qquad (20)$$

So these terms could be written as:

$$\dot{\psi}_{th} = \frac{\tau\exp(-4Sd)}{[1-\exp(-4Sd)]} + \frac{\ln(-1+\tau\exp(-4Sd))}{[1-\exp(-4Sd)]} - \frac{[\tau+\ln(-1+\tau)]}{[1-\exp(-4Sd)]} \qquad (21)$$

$$\dot{\psi}_{fr} = -E\frac{\ln(\exp(-4Sd))}{[1-\exp(-4Sd)]} + E\frac{\ln(-1+\tau\exp(-4Sd))}{[1-\exp(-4Sd)]} - \frac{E\ln(-1+\tau)}{[1-\exp(-4Sd)]} \qquad (22)$$

In which thermal dimensionless entropy is the entropy which is produced due to the temperature difference and the frictional term is also produced due to the friction along the duct.
As we are considering the flow to be fully developed turbulent flow we can use Nu=4.66 and by assuming Glycerol to be the fluid, constants would be calculated. The properties of the fluid are shown in the following table:

C_p	(J/kg.k)	2428
k	(w/m.k)	0.264
ρ	(Kg/m 3)	1260
μ	(Ns/m)	1.48
T_i	(k)	293
T_w	(k)	373
U	(m/s)	0.5,0.1
D	(m)	0.0254

Table 1, Thermodynamic properties of glycerol

Results

And we can plot dimensionless entropy vs. Sd number by assuming E and τ to be constant to find the minimum point for the total entropy generation (fig.2)

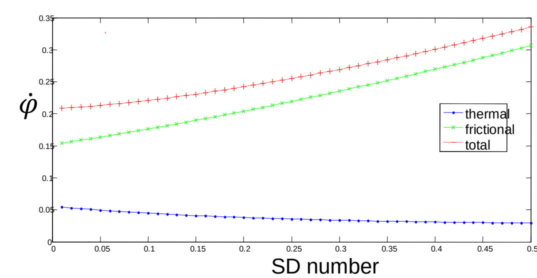

Figure 1, Variation of Dimensionless entropy with SD number τ=0.214

And dimensionless Entropy vs. τ (Fig.3)

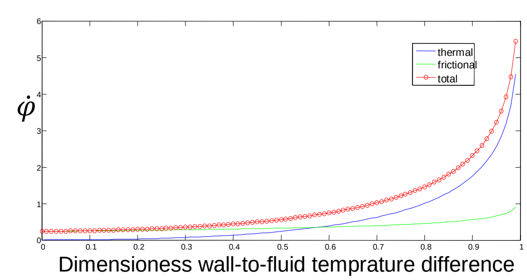

Figure 2, Variation of Dimensionless entropy with τ number Sd=0.39

And plotting the total entropy vs. Sd number for E=0.12 &E=0.007 would lead to (Fig.4)

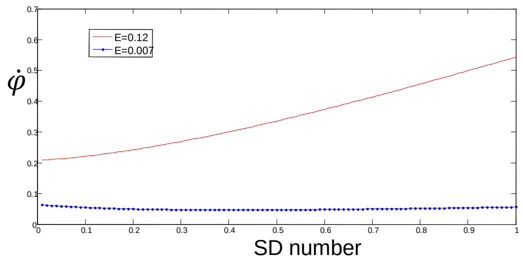

Figure 3, Variation of Dimensionless entropy with SD number, E=0.12 &E=0.007

For $\tau = 0.38$ (Fig.5):

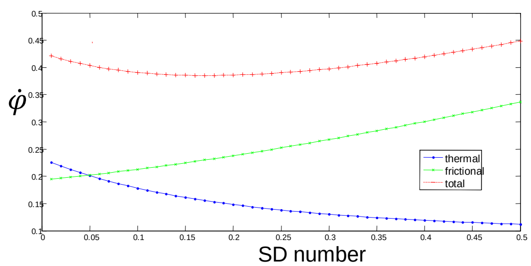

Figure 4, Variation of Dimensionless entropy with SD number, τ=0.38

As it is seen in fig. 2 there is no minimum point for $\tau = 0.214$ but as the τ number is increased the range of the dimensionless thermal entropy generation term starts to grow while the frictional term doesn't change that much. These unbalanced changes in the entropy generation terms lead to the appearance of the minimum point for the total dimensionless entropy generation along the tube for a τ number of approximately $\tau = 0.38$.

Conclusions

This study accords with previous studies that in any heat transfer application with the constant wall temperature boundary condition, the amount of St.L / D, (Sd) is an important design criterion and should be set at the optimum value. In this study the total dimensionless Entropy generation was determined as a function of three dimensionless numbers (Sd, E,τ). The variation of total Entropy generation with any of these three dimensionless numbers has been plotted. It has been shown that there is a minimum point for Entropy generation which leads to the optimum point for the design of the thermodynamic/heat transfer system that yields the maximum efficiency for the system. This point, in a turbulent flow, exists for certain ranges of τ number which starts at approximately $\tau = 0.38$, and for $\tau = 0.21$ there is no minimum for the system's total entropy (in contrast with the laminar flow [6]) and the other parameter, E, has a negligible effect on the existence of an optimum point. Plotting the variations of different terms of the dimensionless entropy generation with the variation of τ number showed that an increase in τ number has a considerable effect on the thermal entropy generation term while not being so influential on the variation of the frictional term.

References

1. Bejan A., Second-law analysis in heat transfer and thermal design, *Adv. Heat Transfer* 15 (1982) 1-58
2. Bejan A., Entropy Generation Minimization, CRC Press, Boca Raton, FL., 1996.
3. A. Z. Sahin, Thermodynamics of laminar viscous flow through a duct subjected to constant heat flux, *Energy*12 (1996) 1179-1187.
4. A. Z. Sahin, Entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature , *International Journal of Heat and Mass Transfer* 43 (2000) 1469-1678.
5. H. Perez-Blanco, Irreversibility in heat transfer enhancement, in: Second Law Aspects of Thermal Design, ASME HTD, Also Presented at The 22nd National Heat Transfer Conference and Exhibition, Niagara Falls, New York, August, 5-8, 1984, vol. 33, pp. 19-26.
6. Esfahani J.A. ,Alizadeh A. (2003), Thermodynamic optimization of laminar convective heat transfer through a pipe with constant wall temperature, Proceedings of the First International Exergy, Energy and Environment Symposium ,Izmir , Turkey, pp.77-83