Optimal Power Control in Spectrum-Sharing Multiple-Access Fading Channels

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***Abstract*—In cognitive radio systems the primary users' spectrum is shared by secondary users. To ensure primary users' reliable communication, total interference inflicted at the primary receivers by secondary transmitters should be lower than a certain interference threshold. In fading environments secondary users can take advantage of the fade property of the channel to opportunistically transmit at high rates at the instants when the channel between the secondary transmitter and the primary receiver is in deep fade so that the interference inflicted at the primary user's receiver remains less than the interference threshold. In this paper, we assume that a number of secondary users are trying to transmit their data to a single secondary receiver in a multiple-access fading channel in presence of primary transmitters and receivers. Here also we use the fade property of the channel to propose a power control policy for maximizing the capacity region of this multiple-access channel. We investigate the problem for two scenarios of having and not having power constraints imposed on secondary transmitters. In both scenarios a power control algorithm is proposed to maximize the capacity region. For the simple case of having just one pair of primary transmitter and receiver without any power constraint imposed on the secondary transmitters we show that the best strategy to maximize the capacity region is to choose the secondary user with the largest ratio of the gain of the channel between this user's transmitter and the secondary receiver to the gain of the channel between this user's transmitter and the primary receiver and allow just this user to transmit while the others are quiet. If the number of primary transmitters and receivers becomes more, while considering the gains of the secondary users channels, we should also include all of the gains of the channels between the secondary transmitters and the primary receivers in our strategy to maximize the capacity region. All of these results are independent of the channels fading distribution.**

***Index Terms*—Cognitive Radio, Spectrum-Sharing, multiple-access channel, Optimal Power Control, Capacity, Fading Channel.**

I. Introduction

Rapid development of communication technology and the widespread use of wireless communication systems have caused a great amount of demand for the radio spectrum which is a very limited and expensive natural resource. By paying money users can buy frequency bands and then they are exclusively licensed to use those bands within an allowed transmit power (to avoid interference). This traditional approach seems to reach to an end since all the limited spectrum is almost allocated. On the other hand studies show that most of the allocated spectrum experiences low utilization in most of the time but unlicensed bands of the spectrum experience heavy utilization [1] and [2]. The idea of cognitive radio was first introduced by J. Mitola [3] to utilize the spectrum efficiently. Comprehensive overviews of the fundamental limits, policy issues, challenges and techniques of cognitive radio can be found in [4]-[7].

In the early cognitive radio papers the secondary users were supposed to sense the bands and when they found spectrum holes they began sending on those bands [7]. Nowadays, the trend is toward considering both of the primary and secondary users transmission simultaneously [8]. In this new scheme it is supposed that the interference inflicted at the primary receivers is below a certain threshold which is named *interference temperature* by [1]. With this assumption the power emitted from a secondary transmitter does not have to be limited as long as the interference inflicted on the primary receiver is below the threshold but there may be power constraints on the secondary transmitters due to some practical considerations like power constraints due to safety considerations or hardware capabilities. Utilizing the concept of interference temperature, recent trend in spectrum-sharing context is on using the constraint on just the received power at the primary receiver while the previous works assumed constraints to be on the transmitted power [9].

Gapstar [10] obtained the channel capacity under received power constraints and gave rise to Ghasemi [9] to find the capacity of fading channels under such constraints. Surprisingly Ghasemi found that the capacity of severe fading channels can become even greater than the capacity of AWGN channels while [11] had previously shown that with transmitted power being constrained instead, capacity is lower than fading (except at low signal-to-noise ratios).

Sum throughput of a multiple-access channel under average transmitted power constraints is on the other hand shown to be maximized when at each instant only the user having the best channel gain is allowed to transmit [12]. This Idea excited us to extend their results to the spectrum-sharing case when a number of secondary transmitters in a multiple-access fading channel with constraints on both the transmitted powers and received powers are trying to send their data to a single receiver.

In order to be sure that we can use the multiple-access channel relations, we model the interference inflicted at the secondary receiver by primary transmitter as weak interference in which we suppose that all the inflicted interference is so weak that we can consider it as a part of the additive noise.

We investigate the problem of maximizing the capacity region of the secondary users' multiple-access channel and propose power control policies to maximize this capacity region. We have solved the problem for the two scenarios of imposing and not imposing power constraints on secondary transmitters. Our results suggest that in order to maximize the capacity region in the simple case of having just one pair of primary transmitter and receiver and not imposing constraints on transmitter powers we should choose the secondary transmitter which has the largest ratio of channel gain between that transmitter and the secondary receiver to the channel gain between that transmitter and the primary receiver and allow just this transmitter to transmit its data while the others are quiet. If the number of primary transmitters and receivers becomes more, while considering the gains of the secondary users channels, we should also include all of the gains of the channels between the secondary transmitters and the primary receivers in our strategy to maximize the capacity region.

The structure of the paper in remainder is as follows. In the next section we describe the system model and our assumptions. The optimal power control policy to reach the maximum capacity region for both scenarios of power constraint in inspected in the sections 3 and 4. Numerical results are shown in the section 5 and concluding remarks are provided in section 6.

II. System Model

In this paper we suppose that in the presence of M primary transmitters and receivers, K secondary transmitters are trying to send their data to a single secondary receiver as shown in Fig. 1. We assume that all the channels between transmitters and receivers are flat fading channels with an arbitrary fading distribution and the noise in the channels is assumed to be white Gaussian with power spectral density N_0 the system bandwidth is also assumed to be B. In the next sections we will show that our results are independent of the fading distribution of the channel . We denote the instantaneous channel power gain between the i-th secondary transmitter and the secondary receiver by g_{ss_i} and the instantaneous channel power gain between the i-th secondary transmitter and the j-th primary receiver by $g_{s_ip_j}$. All of theses gains are assumed to be stationary and ergodic and it is proposed that these random variables have the joint probability density function of $f(g)$ in which $g = (g_{ss1}, g_{ss2}, ..., g_{ssK}, g_{s_1p_1}, g_{s_1p_2}, ..., g_{s_Kp_M})$ is the vector of instantaneous channel power gains. In the numerical results section we have assumed a Rayleigh fading distribution for channels and therefore the channel power gains will have an exponential distribution. Also in that section in order to alleviate complexity of computation we have assumed to have independent fading channels which means that the probability density function $f(g)$ can be factorized in the form $f(g) = \prod_{i=1}^{K} f(g_{ss_i}) \left(\prod_{j=1}^{M} f(g_{s_ip_j})\right)$. Therefore in that section all the channels have exponential channel power gains which are independent and without loss of generality we can assume these gains to have unit mean. Power control

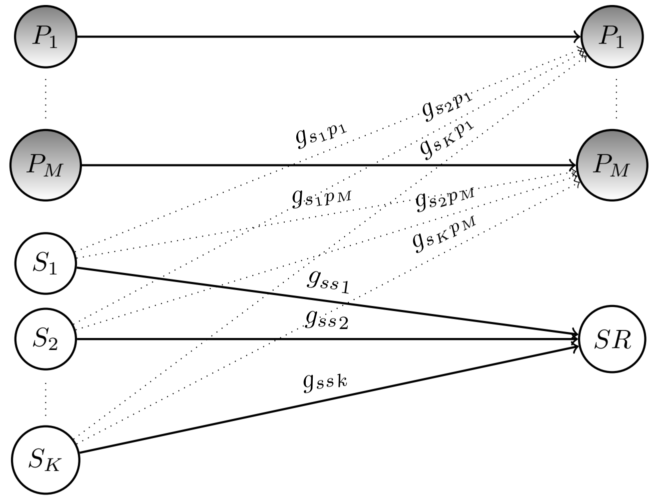

Fig. 1. Multiple-access fading channel with spectrum-sharing interference constraint

at the transmitters requires to know CSI. Thus throughout the paper the secondary transmitters are assumed to have perfect CSI which means that in each instant all the transmitters know the instantaneous value of the vector g which is quite different from the full CSI in [11]. Feeding back the value of g to secondary transmitters may be carried out directly or indirectly through a *band manager* which mediates between the two parties [4]. Using the perfect CSI we suppose that the i-th secondary transmitter sets its power to $P_i(g) = P_i(g_{ss1}, g_{ss2}, ..., g_{ssK}, g_{s_1p_1}, g_{s_1p_2}, ..., g_{s_Kp_M})$. Our objective is to find optimal $P_i(g)$ for $1 \leq i \leq K$ to maximize the capacity region of the multiple-access channel with power control. As stated earlier in spectrum-sharing model the interference inflicted at the primary users receiver should not exceed a certain threshold, Q. The transmitted power of the secondary transmitters can also be constrained to a power P or can be released both of which are studied next.

III. Optimal Power Control With Power Constraint

The interference inflicted at the j-th primary receiver is in fact the sum of interferences by all of the secondary transmitters and thus the interference constraints can be represented as

$$\sum_{i=1}^{K} \int_g g_{s_ip_j} P_i(g) f(g) \mathrm{d}g \leq Q_j \qquad 1 \leq j \leq M \tag{1}$$

In which Q_j is the interference threshold for the j-th primary receiver. By imposing power constraints on secondary transmitters we can suppose that the mean transmitted power of the i-th secondary transmitters should be below the threshold P_i. This can be shown mathematically as

$$\int_g P_i(g) f(g) \mathrm{d}g \leq P_i \qquad 1 \leq i \leq K \tag{2}$$

In fact we want to maximize the multiple-access channel capacity region subject to these two conditions. The capacity region of the multiple-access channel with power control is stated by the following set of equations.

$$\forall S \subset \{0, 1, 2, ..., K\}$$
$$\sum_{i \in S} R_i \leq \frac{1}{2} \log \left(1 + \sum_{i \in S} \frac{P_i(g) g_{ss_i}}{BN_0} \right) \tag{3}$$

In a fading environment the coefficients g_{ss_i} are modelled as random variables and therefore for a fading channel sum-of-rates capacity is the average of the sum-of-rates capacity for each possible realization of the vector g which is stated in (4).

$$C = \frac{1}{2} \int_g \log \left(1 + \sum_{i=1}^{K} \frac{P_i(g) g_{ss_i}}{BN_0} \right) P_i(g) \mathrm{d}g \tag{4}$$

Our capacity maximization problem is now transformed to maximizing (4) subject to (1) and (2) and also another constraint which is the positiveness of power

$$P_i(g) \geq 0 \tag{5}$$

Using the Lagrange multipliers Λ_j for $1 \leq j \leq M$ corresponding to each constraint in (1) and λ_i for $1 \leq i \leq K$ corresponding to each constraint in (2) and using the convexity of the logarithm, we obtain the following system of inequalities governing $P_i(g)$

$$1 + \sum_{i=1}^{K} \frac{P_i(g) g_{ss_i}}{BN_0} \geq \frac{\frac{g_{ss_i}}{BN_0}}{\sum_{j=1}^{M} \Lambda_j g_{s_i p_j} + \lambda_i} \tag{6}$$

With equality if and only if $P_i(g) > 0$.

In order to reduce the complexity of problem we assumed that all of the secondary users have the same average transmit power P and also all the primary receivers have the same interference threshold Q. Therefore by symmetry in inequalities of relation (2), all the λ_is must be equal (we suppose that all of them are equal to the constant λ.) Assuming that the g_{ss_i} and $g_{s_i p_j}$ are all different, we have that $P_i(g) \neq 0 \Rightarrow P_j(g) = 0, \forall j \neq i$ and consequently

$$\frac{\frac{g_{ss_i}}{BN_0}}{\sum_{j=1}^{M} \Lambda_j g_{s_i p_j} + \lambda} \geq \frac{\frac{g_{ss_j}}{BN_0}}{\sum_{j=1}^{M} \Lambda_j g_{s_i p_j} + \lambda} \tag{7}$$

Interpreting this equation we can say that in order to have the maximum sum-of-rates capacity we should choose the secondary transmitter for which the value of direct channel gain to side channel gains Ratio which we denote by $R(i)$ for the i-th secondary transmitter and define by

$$R(i) = \frac{g_{ss_i}}{\sum_{j=1}^{M} \Lambda_j g_{s_i p_j} + \lambda} \tag{8}$$

is largest and allow just this transmitter to transmit with the power that we will find later at any given instant and others must remain quiet until for one of them this value becomes the strongest. In equation (8) every side channel gain in the denominator is multiplied in one of the Lagrange multipliers which are found latter. This shows that each channel gain to some extent contributes in the computation of $R(i)$. If a channel between the secondary transmitter and a primary receiver has a large gain compared to the direct secondary channel (between secondary transmitter and receiver), it can significantly reduce the value of $R(i)$ for that transmitter and vice versa.

If the fading distribution of the channels between the secondary transmitters and primary receivers are all the same which is usually the case and having the assumption of the same interference thresholds, the inequalities of relation (1) would all be the same and therefore all of the Λ_js would be equal to a constant Λ and (8) can be further simplified as

$$R(i) = \frac{g_{ss_i}}{\Lambda \sum_{j=1}^{M} g_{s_i p_j} + \lambda} \tag{9}$$

After doing the optimization processes optimal power control for the i-th secondary transmitter in this case is obtained to be of the form

$$P_i(g) = \frac{1}{\Lambda \sum_{j=1}^{M} g_{s_i p_j} + \lambda} - \frac{BN_0}{g_{ss_i}} \tag{10}$$

For the time when we have

$$\frac{g_{ss_i}}{\Lambda \sum_{j=1}^{M} g_{s_i p_j} + \lambda} > \frac{g_{ss_j}}{\Lambda \sum_{j=1}^{M} g_{s_i p_j} + \lambda} \quad \forall j \neq i \tag{11}$$

and in the same time

$$\frac{g_{ss_i}}{BN_0} > \Lambda \sum_{j=1}^{M} g_{s_i p_j} + \lambda \tag{12}$$

Otherwise the optimal power control is of the form

$$P_i(g) = 0 \tag{13}$$

Now If $R(m) \geq R(i)$ for all $1 \leq i \leq K$ then using the above expressions λ and Λ are found by solving the following system of equations

$$\int_g \left(\frac{1}{\Lambda \sum_{j=1}^{M} g_{s_m p_j} + \lambda} - \frac{BN_0}{g_{ss_m}} \right)^+ f(g) \mathrm{d}g = P \tag{14}$$

$$\int_g g_{s_m p_j} \left(\frac{1}{\Lambda \sum_{j=1}^{M} g_{s_m p_j} + \lambda} - \frac{BN_0}{g_{ss_m}} \right)^+ f(g) \mathrm{d}g = Q \tag{15}$$

In these equations and in the next parts of the paper $(.)^+$ denotes $max\{., 0\}$.The maximum sum-of-rates capacity can then be found by substituting (10) and (13) into (4) as follows

$$C_{pc,max} = \frac{1}{2} \int_g \log \left(\frac{g_{ss_m}}{BN_0 \Lambda \sum_{j=1}^{M} g_{s_m p_j} + BN_0 \lambda} \right) f(g) \mathrm{d}g \tag{16}$$

The above Integration should be taken over the region $g_{ss_m} \geq BN_0 \Lambda \sum_{j=1}^{M} g_{s_m p_j} + BN_0 \lambda$. An important point is that as our proposed algorithm suggests in order to have maximum sum-of-rates capacity, we should search for the secondary transmitter with the largest value of $R(i)$. Also it is notable that the value of $R(i)$ becomes larger if the value of $\frac{g_{ss}}{\sum_{j=1}^{M} g_{sp_j} + \frac{\lambda}{\Lambda}}$ becomes larger which is simply the value of the fraction of instantaneous channel power gain between a secondary transmitter and the secondary receiver to the instantaneous channel power gains between secondary transmitters and the primary receivers plus a constant value. This constant value which is the ratio $\frac{\lambda}{\Lambda}$ becomes smaller if the transmitted power constraint is much more released relative to the interference power constraint. It is interesting to note that our proposed algorithm is a novel type of multiple access technique in which time sharing is performed based on the measurements of the instantaneous channel gains. As it is seen we did not use the probability distribution of $P(g)$ in any parts of our maximization procedure and therefore our results are independent of the fading distribution of the channel.

IV. Optimal Power Control Without Power Constraint

In this section we study the condition in which no power constraint is imposed on the secondary transmitters. In fact this is the case which is now much more considered in the literature these days since satisfying the interference condition in most of the cases causes the power constraints to be satisfied and therefore the power emitted from a secondary transmitter does not have to be limited as long as the interference inflicted on the primary receiver is below the threshold. In this scenario the inequalities of relation (2) no longer exist and therefore we have similar relations to the relations of (6) through (16) but with this difference that in this case we do not have the constant λ in these relations. As a simple case of this scenario we study the case of having just one pair of transmitter and receiver. Here We just have equation (1) which with the assumption of having just one pair of transmitter and receiver introduces the constraint λ_0 in the Lagrange method. The Capacity maximization problem in this scenario reduces to maximizing (4) with the constraints of (1) and (5). After solving this convex optimization problem, (6) reduces to

$$1 + \sum_{i=1}^{K} \frac{P_i(g) g_{ss_i}}{BN_0} \geq \frac{g_{ss_i}}{BN_0 \lambda_0 g_{sp_i}} \tag{17}$$

which should be satisfied for all $1 \leq i \leq K$. The equality is achieved if and only if $P_i(g) \geq 0$. Similar to the scenario discussed in the previous section we see that if $P_i(g) \neq 0 \Rightarrow P_j(g) = 0, \forall j \neq i$ and consequently

$$\frac{g_{ss_i}}{g_{sp_i}} \geq \frac{g_{ss_j}}{g_{sp_j}} \tag{18}$$

Which surprisingly mean that in the case of imposing just the interference condition and ignoring the transmit power constraints the sum-of-rates capacity is maximized when we allow just the secondary transmitter with the largest ratio of the instantaneous power gain of the channel between the secondary transmitter and the secondary receiver to the instantaneous power gain of the channel between the secondary transmitter and the primary receiver to transmit and the other secondary transmitters remain quiet. Optimal power control for the i-th secondary transmitter in this case is of the form

$$P_i(g) = \frac{1}{\lambda_0 g_{sp_i}} - \frac{BN_0}{g_{ss_i}} \tag{19}$$

For the time when we have

$$\frac{g_{ss_i}}{g_{sp_i}} > \frac{g_{ss_j}}{g_{sp_j}} \quad \forall j \neq i \tag{20}$$

and in the same time

$$\frac{g_{ss_i}}{g_{sp_i}} > BN_0 \tag{21}$$

Otherwise the optimal power control is of the form

$$P_i(g) = 0 \tag{22}$$

In this scenario, after finding the optimal power control we can substitute it in the interference constraint to find the accurate value of λ_0. If the index of the secondary transmitter with the largest $\frac{g_{ss_i}}{g_{sp_i}}$ is denoted by m then λ_0 is easily found by solving

$$\int_g \left(\frac{1}{\lambda_0 BN_0} - \frac{g_{ss_m}}{g_{sp_m}} \right)^+ f(g) \mathrm{d}g = \frac{Q}{BN_0} \tag{23}$$

Then the maximum of sum-of-rates capacity is found by substituting the optimal power control into (4) which is

$$C_{npc,max} = \frac{1}{2} \int_g \log \left(\frac{g_{ss_m}}{g_{sp_m} \lambda_0 BN_0} \right) f(g) \mathrm{d}g \tag{24}$$

The integration is taken over the region $\frac{g_{ss_m}}{g_{sp_m}} \geq \lambda_0 BN_0$. As also stated earlier we can see that the transmission scheme is independent of the fading distributions of the channels.

V. Numerical Results

In this section due to the complexity of solving integral equations we present some of the numerical results for the relatively simple case of having just one pair of primary transmitter and receiver. We suppose to have just two secondary transmitters which are trying to communicate with a secondary receiver. We investigate the common case of Rayleigh fading distribution for the channels and suppose that all the instantaneous channel power gains are mutually independent so that we can factorize the probability distribution as

$$f(g) = f_{11}(g_{sp_1}) f_{21}(g_{sp_2}) f_{12}(g_{ss1}) f_{22}(g_{ss2}) \tag{25}$$

Where each f_{ij} is an exponential probability distribution. Without loss of generality we suppose that the mean of all of these probability distributions is unity. We consider the case when there are constraints on transmitted powers and we introduce $\gamma = \lambda BN_0$ and $\Gamma = \Lambda BN_0$. Hence the equation (14) reduces to

$$\int_0^{+\infty} \int_{\Gamma g_{sp_m} + \gamma}^{+\infty} \left(\frac{1}{\Gamma g_{sp_m} + \gamma} - \frac{1}{g_{ss_m}} \right) \times \mathrm{e}^{-g_{ss_m}} \mathrm{e}^{-g_{sp_m}} \mathrm{d}g_{ss_m} \mathrm{d}g_{sp_m} = \frac{P}{BN_0} \tag{26}$$

Computing this double integration, we reach the following equation

$$\frac{\Gamma+1}{\Gamma}\mathrm{e}^{\frac{\gamma}{\Gamma}}\mathrm{Ei}\left(1,\frac{\gamma}{\Gamma}(\Gamma+1)\right)-\mathrm{Ei}(\gamma)=\frac{P}{BN_0} \tag{27}$$

In which $Ei(n,x)$ is an exponential integral defined by

$$\mathrm{Ei}(n,x)=\int_1^{+\infty}\frac{\mathrm{e}^{-xt}}{t^n}\mathrm{d}t \tag{28}$$

Equation (15) also reduces to

$$\int_0^{+\infty}\int_{\Gamma g_{sp_m}+\gamma}^{+\infty}\left(\frac{g_{sp_m}}{\Gamma g_{sp_m}+\gamma}-\frac{g_{sp_m}}{g_{ss_m}}\right) \times\mathrm{e}^{-g_{ss_m}}\mathrm{e}^{-g_{sp_m}}\mathrm{d}g_{ss_m}\mathrm{d}g_{sp_m}=\frac{Q}{BN_0} \tag{29}$$

Using the defined function $Ei(n,x)$ this equation can also be simplified to the equation

$$\mathrm{e}^{\frac{\gamma}{\Gamma}}\left(1-\frac{\gamma}{\Gamma}-\frac{\gamma}{\Gamma^2}\right)\mathrm{Ei}\left(1,\frac{\gamma}{\Gamma}(1+\Gamma)\right) +\frac{\mathrm{e}^{-\gamma}}{\Gamma}-\mathrm{Ei}(\gamma)=\frac{Q}{BN_0} \tag{30}$$

To find the capacity we need to solve the two equations of (27) and (30) simultaneously, to find γ and Γ which is very complicated to do analytically and we should solve these equations numerically. Once we have found these two unknowns we can find the maximum value of sum-of-rates capacity from equation (16). Substituting our probability density function of equation (25) in (16) we have the following integral

$$C_{pc,max}=\frac{1}{2}\int_0^{+\infty}\int_{\Gamma g_{sp_m}+\gamma}^{+\infty}\log\left(\frac{g_{ss_m}}{\Gamma g_{sp_m}+\gamma}\right) \times\mathrm{e}^{-g_{ss_m}}\mathrm{e}^{-g_{sp_m}}\mathrm{d}g_{ss_m}\mathrm{d}g_m \tag{31}$$

We can further simplify this equation using the $Ei(n,x)$ function.

$$C_{pc,max}=-\frac{1}{2}\mathrm{e}^{\frac{\gamma}{\Gamma}}\mathrm{Ei}\left(\frac{\gamma}{\Gamma}(1+\Gamma)\right)+\frac{1}{2}\mathrm{Ei}(\gamma) \tag{32}$$

For the case of having no transmit power constraints the problem is much simpler since we need to solve just one equation. In this case if we substitute the probability density function of (25) into (23) and after simplification we will have

$$\int_{g_{ss_m}}\int_{g_{sp_m}}\left(\frac{1}{\lambda_0 BN_0}-\frac{g_{ss_m}}{g_{sp_m}}\right)f_{m2}(g_{ss_m}) \times f_{m1}(g_{sp_m})\mathrm{d}g_{ss_m}\mathrm{d}g_{sp_m}=\frac{Q}{BN_0} \tag{33}$$

This integration should be taken over the region $\frac{g_{ss_m}}{g_{sp_m}}\geq\lambda_0 BN_0$. By introducing the random variable $w=\frac{g_{ss_m}}{g_{sp_m}}$ we can even simplify this integral as

$$\int_0^{\frac{1}{\lambda_0 BN_0}}\left(\frac{1}{\lambda_0 BN_0}-w\right)f_w(w)\mathrm{d}w=\frac{Q}{BN_0} \tag{34}$$

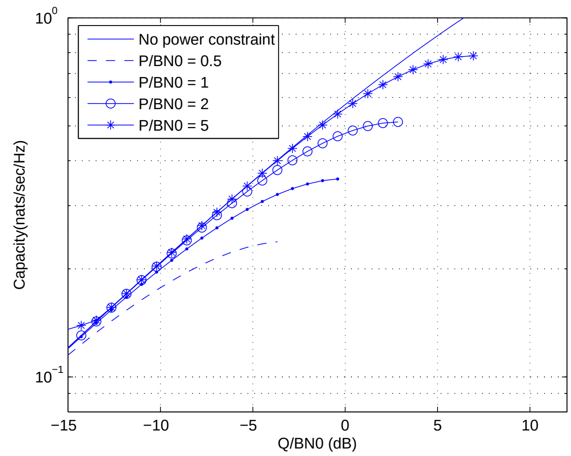

Fig. 2. sum-of-rates capacity versus interference threshold to noise power ratio

Note that the ratio of two exponential random variables of unit mean will have a log-logistic probability density function as defined by

$$f_w(w)=\frac{1}{(1+w)^2},\quad w\geq 0 \tag{35}$$

Therefore after doing the integration using (35), we can even further simplify (34) to

$$\frac{1}{\lambda_0 BN_0}-\log\left(1+\frac{1}{\lambda_0 BN_0}\right)=\frac{Q}{BN_0} \tag{36}$$

We can numerically find λ_0 from this equation and then find the capacity using (24) and (35) as follows

$$C_{np,max}=\frac{1}{2}\int_{\lambda_0 BN_0}^{+\infty}\log\left(\frac{w}{\lambda_0 BN_0}\right)\frac{1}{(1+w)^2}\mathrm{d}w =\frac{1}{2}\log\left(1+\frac{1}{\lambda_0 BN_0}\right) \tag{37}$$

In Fig. 2 we compare the average sum-of-rates capacities for the both cases when we have constraints on transmitted power of the secondary transmitters and when we do not have. This figure clearly shows that when the interference limit is increased the sum-of-rates capacity also increases which was expected. On the other hand when the interference threshold is increased the capacity approaches to the capacity of a simple multiple-access channel and thus at high ratios of Q/BN_0 the capacity seems to become constant as we increase the interference threshold. Also it is easily seen from this figure that by increasing the limiting factor of the secondarys' transmit power P, we approach to the case we do not have any power constraint on transmit power which was expected.

In Fig. 3 we assumed the case when we vary the value of transmit power upper limit on secondary transmitters and evaluated capacity versus interference threshold. As we expected, by increasing the value of P/BN_0 the capacity is also increased since we are removing the power transmission constraints and this causes to increase capacity but for high

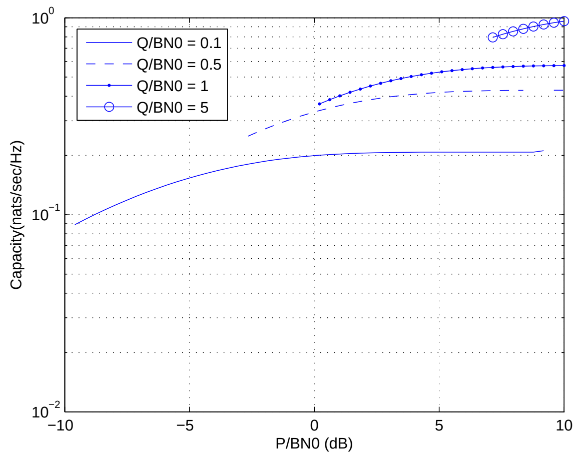

Fig. 3. sum-of-rates capacity versus transmit power limit to noise power ratio

value of P/BN_0 this increase will have less impact on capacity since we have already removed all of the transmit power constraints and we are limited by the interference constraint rather than the transmission power constraints. Also as it is clearly seen from this figure, by increasing the interference threshold Q, the capacity increases since a constraint is being more relaxed. The nonlinear equations of (27) and (30) do not have a solution for every value of P and Q therefore in this figure for every value of Q/BN_0 there exists just a limited region of P/BN_0 for which the equation set has a solution and capacity can be computed.

VI. Conclusion

In spectrum-sharing systems users can use the fade property of the channel to transmit at high rates when the channel between them and the primary receiver is in deep fade. Motivated by this concept we evaluated the multiple-access channel between the secondary users in a spectrum-sharing system in fading environments. For the cases with and without power constraints on secondary transmitters we investigated the problem and we proposed a transmission strategy for the secondary users to achieve the maximum capacity region of the multiple-access channel.

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