Adaptive Fuzzy Sliding Mode Control Using Multiple Models Approach

Nasser Sadati and Rasoul Ghadami
Intelligent Systems Laboratory, Sharif University of Technology, Tehran, Iran
E-mail: sadati@sina.sharif.edu, ghadami@ee.sharif.edu

***Abstract*—In this paper, an adaptive fuzzy sliding mode controller using multiple models approach is presented. By using the multiple models technique the nominal part of the control signal is constructed according to the most appropriate model at different environments. Adaptive single-input single-output (SISO) fuzzy system is used to approximate the discontinuous part of control signal; control gain, in a classical sliding mode controller. The key feature of this scheme is that prior knowledge of the system uncertainties is not required to guarantee the stability. Also the chattering phenomenon in sliding mode control is alleviated and steady tracking error is eliminated. Moreover, a theoretical proof of the stability and convergence of the proposed scheme using Lyapunov method is presented.**

I. INTRODUCTION

Complex systems such as robotic manipulators, because of uncertainties in their structures, are in general very difficult in control. Theses systems conclude uncertainties due to fault in the system, sensor and actuator failure, external disturbance and change in system parameter. A powerful scheme to control the complex systems is multiple models control strategy [3], [4]. Because these systems operate in multiple environments which may change abruptly from one to another, therefore, a good way to improve controller performance is to use multiple models control if models are approximately available for different environments [5]. The objective in multiple models controller is to determine the most appropriate model at any instant, using a suitable performance criterion based on the identification error, and to activate the corresponding controller.

Classical sliding mode controller design provides a systematic approach to the problem of maintaining stability in the face of modeling imprecision and uncertainty. This control scheme utilizes a high speed switching control law to drive the nonlinear predefined state trajectory onto a specified surface, to attain the conventional goals of control such as stabilization and tracking. However, this control scheme has important drawbacks limiting its practical applicability, such as chattering and large control authority. Moreover, in order to guarantee the stability of the sliding mode control systems, the boundary of the uncertainty has to be estimated. However, the estimate of the boundary is difficult to know, thus a conservative control law is selected which deteriorates the system performance.

Recently, much research has been done to use fuzzy logic in order to improve the performance and alleviate the problems met in practical implementation of sliding mode controllers [6]. Also various fuzzy sliding mode controllers have been proposed for robotic manipulators. These works can be divided into two different types. In the first type of fuzzy sliding mode controller, it is assumed that the model of robotic manipulator is totally unknown. For instance in [7] and [8], fuzzy systems are used to implement the system dynamics as well as the control gain. In the second type of fuzzy sliding mode controller, it is assumed that the model of robotic manipulator is partly known with the analysis of the physical properties of the system. Efforts are contributed to the construction of the control gain [9] and [10].

In this paper, adaptive sliding mode control with the combination of fuzzy system and multiple models control are used to control the robotic manipulators. The multiple models approach is used to construct the equivalent control of the sliding mode control signal. The use of multiple models approach offers more robustness and reduces both the control gain authority and tracking error of the transient state. In other side, the discontinuous part of the control signals in the classical sliding mode controllers are substituted by SISO fuzzy systems to eliminate the chattering phenomenon. Also the centers of membership functions in the consequence part of the fuzzy systems are updated in an on-line manner to compensate the system uncertainties and to guarantee the stability of the overall system without having any prior knowledge of the system uncertainties. The adaptive law is designed based on the Lyapunov method and mathematical proof for the stability and convergence of the overall system is provided.

The rest of this paper is organized as follows. Classical sliding mode controller for robotic manipulators is summarized in section II. The structure of classical multiple models approach is described in section III. The adaptive fuzzy sliding mode controller using multiple models approach for robotic manipulators is presented in section IV. The simulation results are given in section V to demonstrate the effectiveness of the proposed control scheme. Finally, section VI presents some concluding remarks.

II. CLASSICAL SLIDING MODE CONTROL FOR ROBOTIC MANIPULATORS

This section briefly reviews the basic concepts of classical sliding mode control for robotic manipulators.

A. Model or Robotic Manipulators

The dynamic equation of an n-link rigid robotic manipulator system is described by the following second order nonlinear vector differential equation

$$\boldsymbol{M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = u} \quad (1)$$

where $\boldsymbol{q}=[q_1,...,q_n]^T$ is an $n\times1$ vector of joint angular position, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are $n\times1$ vectors of corresponding velocity and acceleration, $\boldsymbol{u}$ is an $n\times1$ vector of applied joint torques (control inputs), $\boldsymbol{M(q)}$ is an $n\times n$ inertial matrix, $\boldsymbol{B(q,\dot{q})}$ is an $n\times n$ matrix of Coriolis and centrifugal forces and $\boldsymbol{G(q)}$ is an $n\times1$ gravity vector. The inertial matrix $\boldsymbol{M(q)}$ is symmetric and positive definite. It is assumed that a robotic manipulator, as is described by (1), has some known parts and some unknowns and therefore, there exist uncertainty in the system model. Thus $\boldsymbol{M(q)}$, $\boldsymbol{B(q,\dot{q})}$ and $\boldsymbol{G(q)}$ can be described as

$$\begin{aligned}\boldsymbol{M(q)}&=\boldsymbol{\hat{M}(q)}+\Delta\boldsymbol{M(q)}\\ \boldsymbol{B(q,\dot{q})}&=\boldsymbol{\hat{B}(q,\dot{q})}+\Delta\boldsymbol{B(q,\dot{q})}\\ \boldsymbol{G(q)}&=\boldsymbol{\hat{G}(q)}+\Delta\boldsymbol{G(q)}\end{aligned}\tag{2}$$

where $\boldsymbol{\hat{M}(q)}$, $\boldsymbol{\hat{B}(q,\dot{q})}$ and $\boldsymbol{\hat{G}(q)}$ are the known parts or estimated parameters, and $\Delta\boldsymbol{M(q)}$, $\Delta\boldsymbol{B(q,\dot{q})}$ and $\Delta\boldsymbol{G(q)}$ are the unknown parts. For simplification in notation, we avoid writing the variables in the parentheses of the above matrices and vectors from now on.

B. *Classical Sliding Mode Control for Robot Manipulator*

In the design of sliding mode controller for robotic manipulators, the control objective is to drive the joint position $\boldsymbol{q}$ to the desired position $\boldsymbol{q_d}$. So by defining the tracking error to be in the following form

$$\boldsymbol{e}=\boldsymbol{q}-\boldsymbol{q_d}\tag{3}$$

the sliding surface can be written as

$$\boldsymbol{s}=\dot{\boldsymbol{e}}+\boldsymbol{\lambda e}\tag{4}$$

where $\lambda=diag[\lambda_1,...,\lambda_i,...,\lambda_n]$, in which λ_i is a positive constant. The control objective can now be achieved by choosing the control input so that the sliding surface satisfies the following sufficient condition

$$\frac{1}{2}\frac{d}{dt}s_i^{\,2}\le-\eta_i|s_i|\tag{5}$$

where η_i is a positive constant. Equation (5) indicates that the energy of $\boldsymbol{s}$ should decay as long as $\boldsymbol{s}$ is not zero. To set up the control $\boldsymbol{u}$, define the reference states to be

$$\begin{aligned}\dot{\boldsymbol{q}}_{\boldsymbol{r}}&=\dot{\boldsymbol{q}}-\boldsymbol{s}=\dot{\boldsymbol{q}}_{\boldsymbol{d}}-\boldsymbol{\lambda e}\\ \ddot{\boldsymbol{q}}_{\boldsymbol{r}}&=\ddot{\boldsymbol{q}}-\dot{\boldsymbol{s}}=\ddot{\boldsymbol{q}}_{\boldsymbol{d}}-\boldsymbol{\lambda}\dot{\boldsymbol{e}}\end{aligned}\tag{6}$$

Now the control input $\boldsymbol{u}$ can be chosen to be in the following form

$$\begin{aligned}\boldsymbol{u}&=\hat{\boldsymbol{u}}-\boldsymbol{As}-\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})\\ \hat{\boldsymbol{u}}&=\hat{\boldsymbol{M}}\ddot{\boldsymbol{q}}_{\boldsymbol{r}}+\hat{\boldsymbol{B}}\dot{\boldsymbol{q}}_{\boldsymbol{r}}+\hat{\boldsymbol{G}}\end{aligned}\tag{7}$$

where $\boldsymbol{K}=diag[k_{11},...,k_{ii},...,k_{nn}]$ is a diagonal positive definite matrix in which k_{ii}'s are positive constants and $\boldsymbol{A}=diag[a_1,...,a_i,...,a_n]$ is a diagonal positive definite matrix in which a_i's are also positive constants. Putting (7) into (1) leads to

$$\boldsymbol{M\dot{s}}+\boldsymbol{(B+A)s}=\boldsymbol{\Delta f}-\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})\tag{8}$$

where $\boldsymbol{\Delta f}=\boldsymbol{\Delta M}\ddot{\boldsymbol{q}}_{\boldsymbol{r}}+\boldsymbol{\Delta B}\dot{\boldsymbol{q}}_{\boldsymbol{r}}+\boldsymbol{\Delta G}$. It has been proven ([2]) that by considering the Lyapunov function candidate as

$$V=\frac{1}{2}\boldsymbol{s}^T\boldsymbol{Ms}\tag{9}$$

and choosing $\boldsymbol{K}$ such that

$$k_{ii}\ge\left|\Delta f_i\right|_{bound}\tag{10}$$

where $\left|\Delta f_i\right|_{bound}$ is the boundary of $\left|\Delta f_i\right|$, the overall system is asymptotically stable. Therefore the decay of the energy of $\boldsymbol{s}$, as long as $\boldsymbol{s}\neq0$, is guaranteed and the sufficient condition in (5) is satisfied.

III. Classical Multiple Models Control

As stated before, the multiple models control strategy is to determine the best model for the current environment at every instant and to apply the appropriate control signal correspondingly. Therefore it can be divided into two separated phases; identification and control.

A. *Identification*

Identification part is composed of predefined models which cover different environments of the plant and decision mechanism in order to decide about which model is the closest one to the plant. Many techniques are used in identify the best model in multiple models controller [3]-[5]. Classical types of these controllers work based on the switching. Switching action is determined in decision mechanism part and in which the model with the smallest error, according to some criterion, is chosen rapidly. If y and $\hat{y}_n$ is considered as the output of the plant and the nth model, respectively, One specific performance index is chosen as follow

$$J_n(t)=a\,e_n^{\,2}(t)+b\int_0^t e^{-\kappa(t-\tau)}e_n^{\,2}(\tau)\,d\tau\tag{11}$$

$$a\ge0\quad b,\kappa>0$$

where

$$e_n(t)=y(t)=\hat{y}_n(t)\tag{12}$$

a and b are the weighting factors on the instantaneous measure and the long term accuracy. κ is a forgetting factor which makes previous measures have different weightings and as time goes on, old values have less effect on the result.

B. *Control*

The control problem in the multiple models technique is preformed in two steps: generation of the basic control signal for each model separately and switching to the best controller according to the selected model. In classical multiple models approach N separated controllers are designed for predefined models so that the control objective is satisfied for each of them. The final control is determined among N calculated control signals based on the best selected model in the control mechanism at each instant of time.

IV. ADAPTIVE FUZZY SLIDING MODE CONTROL USING MULTIPLE MODELS APPROACH

There are major disadvantages in designing the classical sliding mode controllers. First, because of the control actions which are discontinuous across $\boldsymbol{s}$, there is chattering in a boundary of the surface $\boldsymbol{s}$. Such high frequency switching (chattering) might excite unmodeled dynamics and impose undue wear and tear on the actuators, so the control law would not be considered acceptable. Second, the prior knowledge of the boundary of uncertainty is required in compensators. If boundary is unknown, a large value has to be applied to the gain of discontinuous part of control signal and this large control gain may intensify the chattering on the sliding surface.

In this section, an adaptive fuzzy sliding mode controller using multiple models approach has been proposed to avoid the aforementioned problems. For this purpose, first adaptive SISO fuzzy system, as continuous function, is applied to calculate each elements of control gain $\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})$ in the control law (7). The control input is written as

$$\boldsymbol{u} = \hat{\boldsymbol{u}} - \boldsymbol{A}\boldsymbol{s} - \boldsymbol{K} \tag{13}$$

where $\boldsymbol{K} = [k_1, \ldots, k_i, \ldots, k_n]^T$ is an $n \times 1$ vector in which k_i is the output of the ith fuzzy system.

Because the control gain in (7) has the sane sign as s_i, therefore k_i should have the same sign as s_i. When $|s_i|$ is large, it is expected that $|k_i|$ is also large and when $|s_i|$ is small, $|k_i|$ is small to avoid chattering. This idea similar to applying the function $sat(.)$. The difference is that the control gain varies along with the sliding surface all the time. In addition, the adaptive law is designed to guarantee that the k_i can compensate the system uncertainty. Thus, the fuzzy system for k_i should be SISO, with s_i as the input.

There are usually four basic parts in a fuzzy system; fuzzification, fuzzy rule base, fuzzy inference engine and defuzzification. The rules in the rule base are in the following format:

$$\text{IF } s_i \text{ is } A_i^m \text{, THEN } k_i \text{ is } B_i^m \tag{14}$$

where A_i^m and B_i^m are fuzzy sets. In this paper, it is chosen that both s_i and k_i have the same kind of membership functions : *NB* , *NM , NS, ZE, PS, PM, PB*, where *N* stands for negative, *P* positive, *B* big, *M* medium, *S* small and *ZE* zero. They are all Gaussian membership functions defined as

$$\mu_A(x_i) = \beta \exp\left[-\left(\frac{x_i - \theta}{\sigma}\right)^2\right] \tag{15}$$

where "*A*" represents one of the fuzzy sets *NB, ... , PB* and x_i represents s_i or k_i. β is a positive constant. θ and σ are the center and the width of "*A*" respectively. Although the membership functions for s_i and k_i have the same titles, correspondingly, the values of the center and the width of the membership function with a same title for s_i and k_i are different. The parameters of k_i the membership functions of s_i are pre-defined, while those of k_i are updated on-line.

Based of the above discussions and the definitions of the input and output membership functions, the rule base can be decided as follow

IF s_i is NB, THEN k_i is NB
IF s_i is NM, THEN k_i is NM
IF s_i is NS, THEN k_i is NS
IF s_i is ZE, THEN k_i is ZE
IF s_i is PS, THEN k_i is PS
IF s_i is PM, THEN k_i is PM
IF s_i is PB, THEN k_i is PB (16)

By choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine, k_i can be written as

$$k_i = \frac{\sum_{m=1}^{M} \theta_{k_i}^m \, \mu_{A^m}(s_i)}{\sum_{m=1}^{M} \mu_{A^m}(s_i)} = \boldsymbol{\theta}_{k_i}{}^T \Phi_{k_i}(s_i) \tag{17}$$

where $\boldsymbol{\theta}_{k_i} = [\theta_{k_i}^1, \ldots, \theta_{k_i}^m, \ldots, \theta_{k_i}^M]^T$ is the $m \times 1$ vector. Also $\Phi_{k_i}(s_i) = [\varphi_{k_i}^1(s_i), \ldots, \varphi_{k_i}^m(s_i), \ldots, \varphi_{k_i}^M(s_i)]^T$ is the $m \times 1$ vector where $\varphi_{k_i}^m(s_i) = \mu_{A^m}(s_i) \Big/ \sum_{m=1}^{M} \mu_{A^m}(s_i)$. $\boldsymbol{\theta}_{k_i}$ is chosen as the parameter to be updated and therefore is called the parameter vector, and $\Phi_{k_i}(s_i)$ can be regarded as the weight of the parameter vector.

As stated, a good way to improve controller performance of complex systems with abrupt change is to use multiple models control if models are approximately available for different environments. Therefore the multiple models technique is used to construct the equivalent term $\hat{\boldsymbol{u}}$ in the control law (13). The control block diagram of the proposed architecture is shown in Fig. 1, where the input of each fuzzy system is s_i and the corresponding output is k_i.

$M_1, M_2, \ldots, M_N$ are estimated models in different environment of robotic action which is used in parallel with the plant. In other words, $\hat{\boldsymbol{M}}_{\boldsymbol{n}}$, $\hat{\boldsymbol{B}}_{\boldsymbol{n}}$ and $\hat{\boldsymbol{G}}_{\boldsymbol{n}}$, for $n = 1, \ldots, N$, are the estimated parameters of the nth environment , and the unknown parts of the plant in each environment is defined as

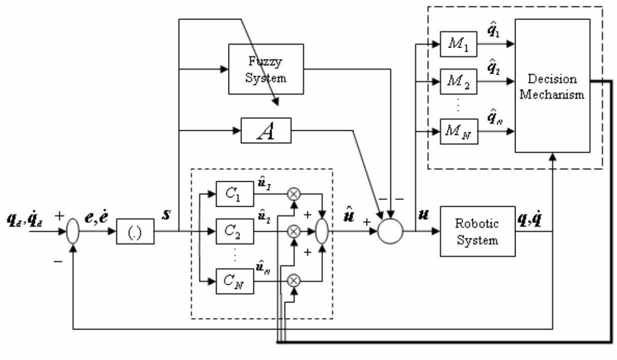

Fig.1 Adaptive fuzzy sliding mode control using multiple models approach

$$\boldsymbol{\Delta M_n} = \boldsymbol{\hat{M}_n} - \boldsymbol{M}$$
$$\boldsymbol{\Delta B_n} = \boldsymbol{\hat{B}_n} - \boldsymbol{B}$$
$$\boldsymbol{\Delta G_n} = \boldsymbol{\hat{G}_n} - \boldsymbol{G} \qquad n = 1,\ldots,N \tag{18}$$

Moreover $\boldsymbol{q}$ and $\boldsymbol{\hat{q}_n}$ are the output of the plant and nth model, respectively. $\boldsymbol{\hat{u}}_n$, for $n = 1,\ldots,N$, is the output of nth local controller corresponding to nth model which is defined as

$$\boldsymbol{\hat{u}}_n = \boldsymbol{\hat{M}}_n \boldsymbol{\ddot{q}_r} + \boldsymbol{\hat{B}}_n \boldsymbol{\dot{q}_r} + \boldsymbol{\hat{G}}_n \tag{19}$$

In identification mechanism part, the performance index is selected as

$$J_n(t) = a \left\| \boldsymbol{\hat{e}_n}(t) \right\| + b \int_0^t e^{-\kappa(t-\tau)} \left\| \boldsymbol{\hat{e}_n}(\tau) \right\| d\tau \tag{20}$$
$$a \geq 0 \quad b, \kappa > 0$$

where

$$\boldsymbol{\hat{e}_n}(t) = \boldsymbol{q}(t) - \boldsymbol{\hat{q}_n}(t) \tag{21}$$

Since we use blending instead of switching between each controller , as seen in Fig. 2, to achieve continuous control signal, the control signal is written by

$$\boldsymbol{u} = \sum_{n=1}^{N} \alpha_n \boldsymbol{\hat{u}}_n - \boldsymbol{As} - \boldsymbol{K} \tag{22}$$

where blending coefficient α_n is defined as

$$\alpha_n(t) = \frac{e^{-|J_n(t)|^2/\delta^2}}{\sum_{n=1}^{N} e^{-|J_n(t)|^2/\delta^2}} \tag{23}$$

The transform uses exponential function and then normalize these values so that α_n's have the property to lie between 0 and 1 , and $\sum_{n=1}^{N} \alpha_n = 1$.

In continue, an adaptive law is designed to guarantee that k_i's can compensate the system uncertainties. Now putting (22) into (1) leads to

$$\boldsymbol{M}\,\boldsymbol{\dot{s}} = -\boldsymbol{(B + A)}\,\boldsymbol{s} + \boldsymbol{\Delta f} - \boldsymbol{K} \tag{24}$$

where $\boldsymbol{\Delta f} = \sum_{n=1}^{N} \alpha_n \boldsymbol{\Delta f_n}$ in which $\boldsymbol{\Delta f}_n = \boldsymbol{\Delta M}_n \boldsymbol{\ddot{q}_r} + \boldsymbol{\Delta B}_n \boldsymbol{\dot{q}_r} + \boldsymbol{\Delta G}_n$.

Now, defining $\boldsymbol{\theta}_{k_{id}}$ so that $k_i = \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)$ is the optimal compensation for Δf_i, according to Wang's theorem [1], there exists $\delta_i > 0$ satisfying

$$\left| \Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i) \right| \leq \delta_i \tag{25}$$

where δ_i is arbitrary and can be chosen as small as possible. Defining

$$\boldsymbol{\tilde{\theta}}_{k_i} = \boldsymbol{\theta}_{k_i} - \boldsymbol{\theta}_{k_{id}} \tag{26}$$

It can be shown that by choosing the adaptive law as

$$\dot{\boldsymbol{\tilde{\theta}}}_{k_i} = s_i \Phi_{k_i}(s_i) \tag{27}$$

the overall system is asymptotically stable with respect to $\boldsymbol{s}$ and the actual joint angular positions converge to the desired ones.

***Proof**:* Choose the Lyapunov candidate as

$$V = \frac{1}{2} \boldsymbol{s}^T \boldsymbol{Ms} + \frac{1}{2} \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \boldsymbol{\tilde{\theta}}_{k_i} \tag{28}$$

where $\boldsymbol{M}$ is symmetric positive matrix and $\boldsymbol{\tilde{\theta}}_{k_i}{}^T \boldsymbol{\tilde{\theta}}_{k_i} > 0$, therefore V is positive definite. Now Consider the derivative of V , given by

$$\begin{aligned}
\dot{V} &= \frac{1}{2}[\boldsymbol{\dot{s}}^T \boldsymbol{Ms} + \boldsymbol{s}^T \boldsymbol{\dot{M}s} + \boldsymbol{s}^T \boldsymbol{M\dot{s}}] + \frac{1}{2} \sum_{i=1}^{n} [\dot{\boldsymbol{\tilde{\theta}}}_{k_i}{}^T \boldsymbol{\tilde{\theta}}_{k_i} + \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i}] \\
&= \frac{1}{2}[\boldsymbol{s}^T \boldsymbol{\dot{M}s} + 2\boldsymbol{s}^T \boldsymbol{M\dot{s}}] + \frac{1}{2} \sum_{i=1}^{n} 2\boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i} \\
&= \boldsymbol{s}^T [\boldsymbol{M\dot{s}} + \boldsymbol{Bs}] + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i} \\
&= \boldsymbol{s}^T [-\boldsymbol{(B + A)s} + \boldsymbol{\Delta f} - \boldsymbol{K} + \boldsymbol{Bs}] + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i} \\
&= -\boldsymbol{s}^T \boldsymbol{As} + \boldsymbol{s}^T [\boldsymbol{\Delta f} - \boldsymbol{K}] + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i} \\
&= -\boldsymbol{s}^T \boldsymbol{As} + \sum_{i=1}^{n} (s_i [\Delta f_i - k_i]) + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i}
\end{aligned} \tag{29}$$

Since $k_i = \boldsymbol{\tilde{\theta}}_{k_i}{}^T \Phi_{k_i}(s_i) + \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)$, then

$$\begin{aligned}
\dot{V} &= -\boldsymbol{s}^T \boldsymbol{As} + \sum_{i=1}^{n} (s_i [\Delta f_i - (\boldsymbol{\tilde{\theta}}_{k_i}{}^T \Phi_{k_i}(s_i) + \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i))]) \\
&\quad + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i} \\
&= -\boldsymbol{s}^T \boldsymbol{As} + \sum_{i=1}^{n} (s_i [\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)]) \\
&\quad + \sum_{i=1}^{n} (-s_i \boldsymbol{\tilde{\theta}}_{k_i}{}^T \Phi_{k_i}(s_i)) + \sum_{i=1}^{n} \boldsymbol{\tilde{\theta}}_{k_i}{}^T \dot{\boldsymbol{\tilde{\theta}}}_{k_i}
\end{aligned}$$

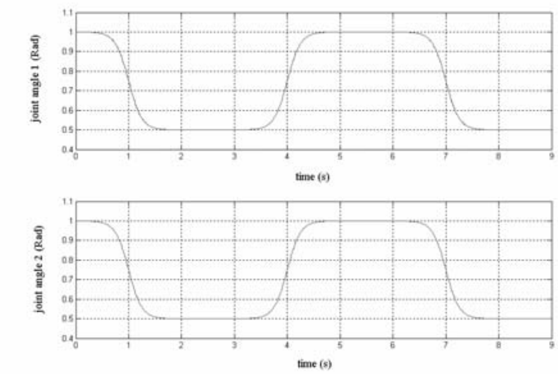

Fig.2 Tracking of joint angles 1 and 2

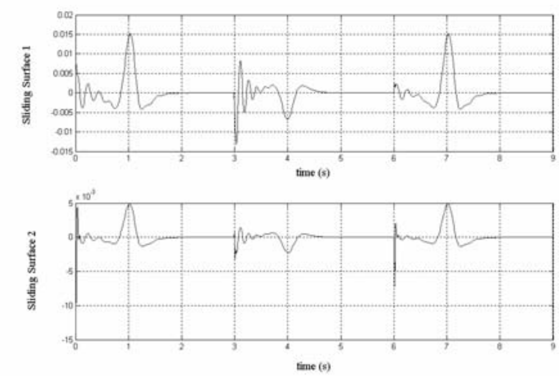

Fig.3 Sliding surface of joints 1 and 2.

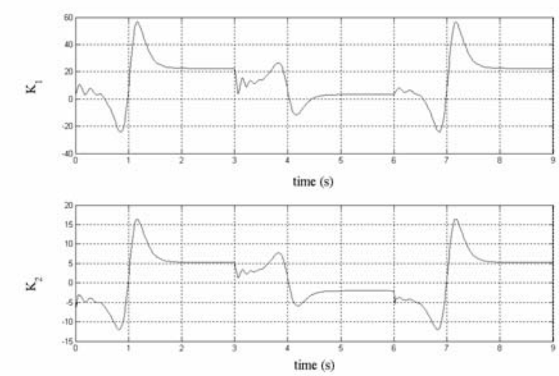

Fig.4 Control gains of joints 1 and 2 (outputs of fuzzy systems).

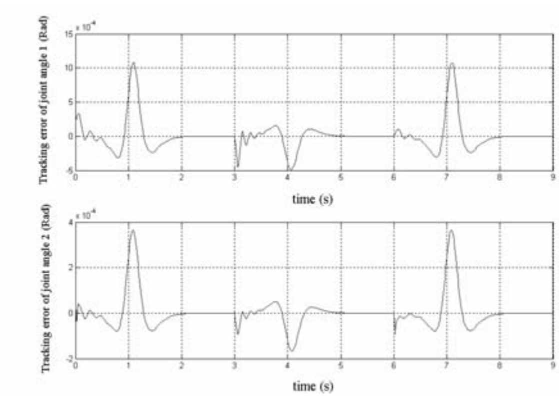

Fig.5 Tracking errors of joint angles 1 and 2.

$$= -\boldsymbol{s}^T \boldsymbol{A}\boldsymbol{s} + \sum_{i=1}^{n} (s_i[\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)])$$

$$+ \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_{k_i}{}^T [-s_i \Phi_{k_i}(s_i)) + \dot{\tilde{\boldsymbol{\theta}}}_{k_i}] \qquad (30)$$

Moreover, since the adaptive law in (27) is chosen as $\dot{\tilde{\boldsymbol{\theta}}}_{k_i} = s_i \Phi_{k_i}(s_i)$, then

$$\dot{V} = -\boldsymbol{s}^T \boldsymbol{A}\boldsymbol{s} + \sum_{i=1}^{n} (s_i[\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)]) \qquad (31)$$

From (25), there exist

$$\left|\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)\right| \le \delta_i \qquad (32)$$

where δ_i can be chosen as small as possible. By assuming

$$\left|\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)\right| \le \delta_i \le \rho_i |s_i| \qquad (33)$$

where $0 < \rho_i < 1$, the second term at the right side of (31) satisfies

$$s_i[\Delta f_i - \boldsymbol{\theta}_{k_{id}}{}^T \Phi_{k_i}(s_i)] \le \rho_i |s_i|^2 = \rho_i s_i{}^2$$

Therefore

$$\dot{V} \le -\boldsymbol{s}^T \boldsymbol{A}\boldsymbol{s} + \sum_{i=1}^{n} \rho_i s_i{}^2$$

$$\le \sum_{i=1}^{n} (-a_i s_i{}^2 + \rho_i s_i{}^2) \qquad (34)$$

Now by simply choosing $a_i > \rho_i$

$$\dot{V} \le \sum_{i=1}^{n} (\rho_i - a_i) s_i{}^2 \le 0 \qquad (35)$$

and $\dot{V} = 0$ only when $s_i = 0$. Thus, the overall system with the adaptive law in (27) is asymptotically stable with respect to $\boldsymbol{s}$. In other words

$$\lim_{t\to\infty} \boldsymbol{s} = \lim_{t\to\infty} (\dot{\boldsymbol{e}} + \boldsymbol{\lambda}\boldsymbol{e}) = 0 \qquad (36)$$

or equivalently

$$\lim_{t\to\infty} \boldsymbol{e} = 0 \ \ \& \ \ \lim_{t\to\infty} \dot{\boldsymbol{e}} = 0 \qquad (37)$$

Therefore, it is proved that the adaptive multiple models sliding mode control input (22) drives the actual joint positions to their desired values. Q.E.D.

V. Simulation Results

In this section, the proposed adaptive multiple models sliding mode controller is used on a two-link robotic manipulator, whose parameter matrices are as follows

$$\boldsymbol{M(q)} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

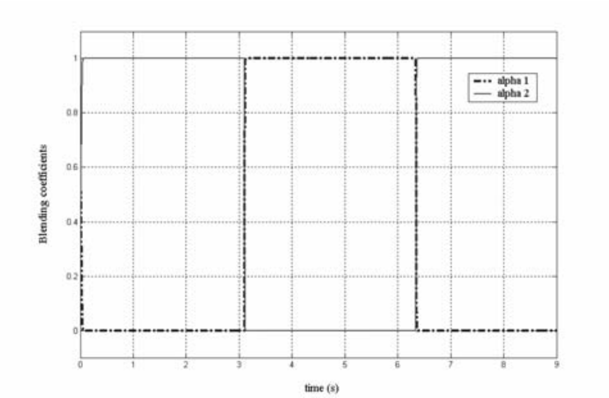

Fig.6 Blending coefficients of control action.

where

$$M_{11} = (m_2 + m_3)l_2^{\ 2} + (m_2 + m_3)l_1 l_2 \cos q_2$$
$$M_{12} = (m_1 + m_2 + m_3)l_1^{\ 2} + (m_2 + m_3)l_2^{\ 2} + 2(m_2 + m_3)l_1 l_2 \cos q_2$$
$$M_{12} = M_{21}$$
$$M_{22} = (m_2 + m_3)l_2^{\ 2}$$

and

$$\boldsymbol{B(q,\dot{q})} = \begin{bmatrix} -2(m_2 + m_3)l_1 l_2 \dot{q}_2 \sin q_2 & -(m_2 + m_3)l_1 l_2 \dot{q}_2 \sin q_2 \\ (m_2 + m_3)l_1 l_2 \dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$\boldsymbol{G(q)} = \begin{bmatrix} (m_1 + m_2 + m_3)gl_1 \cos q_1 + (m_2 + m_3)gl_2 \cos(q_1 + q_2) \\ (m_2 + m_3)gl_2 \cos(q_1 + q_2) \end{bmatrix}$$

where m_1 and m_2 are the masses, and l_1 and l_2 are the lengths of the links 1 and 2, respectively. The values of these parameter are chosen as $m_1 = 2\ kg$, $m_2 = 1\ kg$, $l_1 = 2\ m$ and $l_2 = 1\ m$. m_3 represents the mass of the load at the end of the link 2. The situation that is simulated is as following. The manipulator is expected to take a load from position one at $[1,1]^T\ Rad$ to position two at $[0.5, 0.5]^T\ Rad$ repeatedly. The manipulator moves between positions along a predefined trajectory during 2 sec and stays in position one and two for 1 sec. to take and put the load $m_3 = 2\ kg$ respectively. In this simulation, two models are selected for the situation of $m_3 = 0$ and $m_3 = 2\ kg$, i.e. $N = 2$. Each model is estimated by applying a factor to the corresponding parameter matrices of the original system in each environment to count uncertainties, i.e., $\hat{\boldsymbol{G}}_n = 0.9\boldsymbol{G}$, $\hat{\boldsymbol{B}}_n = 0.8\boldsymbol{B}$ and $\hat{\boldsymbol{M}}_n = 0.95\boldsymbol{M}$ for $n = 1, 2$, using $m_3 = 0$ and $m_3 = 2\ kg$ respectively. Parameters of identification part is chosen as $a/b = 1$, $\kappa = 6$ and $\delta = 0.01$. The control input $\boldsymbol{u}$ is chosen as in (22), where $\boldsymbol{A} = diag[40, 40]$, $\boldsymbol{\lambda} = diag[10, 10]$.

The simulation results are shown in Fig. 2 - Fig. 6. As seen in Fig. 2, the joint angles track the desired trajectories and the proposed control scheme drives the robotic manipulator to its desired positions. Moreover, there is no chattering in the sliding surface as shown in Fig. 3, and also the values of $\boldsymbol{K}$ converge to constant in the steady-state as is shown in Fig. 4 . Also Fig. 5 shows that the tracking errors converge to zero. Finally, Fig. 6 shows the blending coefficients of control action.

VI. Conclusion

In this paper an adaptive fuzzy sliding mode controller using multiple models approach is proposed for robotic manipulators. The equivalent control term of the control signal is constructed by blending the local control action according to the most appropriate model at any environment. In this way the tracking errors of the transient state is reduced considerably. The discontinuous parts of the classical sliding mode controller are replaced by adaptive SISO fuzzy systems to avoid the chattering. The system uncertainties are compensated by the adaptive control gain and therefore the system stability is guaranteed without any prior knowledge of the system uncertainties. Since the fuzzy systems which are used in the controller are SISO systems, therefore the design and implementation of the controller is simplified. Moreover, the stability and convergence of the overall system are proved by the Lyapunov method. The simulation results demonstrate that the proposed adaptive fuzzy sliding mode control using multiple models approach, as proved theoretically, is a stable control scheme for robotic manipulators and works well in complicated situations

References

[1] L. X. Wang, *A Course in Fuzzy Systems and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1997.

[2] K. D. Yong, "Controller design for a manipulator using theory of variable structure system,".*IEEE Trans. Systs., Man, Cybern*., vol. SMC-8, no .2, pp. 210-218, Feb. 1978.

[3] K. S. Narendra, J. Balakrishnan, "Adaptive control using multiple models, " *IEEE Trans. Automatic Control*, vol. 42, no. 2, pp. 171-187, Feb. 1997.

[4] Kumpati. S. Narendra, Cheng xiang, "Adaptive control of discrete-time systems using multiple models," *IEEE Trans. Automatic control*, vol. 45, no. 9, pp. 1669-1686, sept. 2000.

[5] K S. Narendra, J. Balakrishnan, "Improving transient response of adaptive control systems using multiple models and switching," *IEEE Trans. Automatic Control*, vol. 39, pp. 1861-1866, Sept. 1994.

[6] O. Kaynak, K. Erbartur, M. Ertugrul. "The Fusion of computationally intelligent methodologies and sliding mode control-a survey," *IEEE Trans. on Industrial Electronics*, vol. 48, no. 1, pp. 4-17, Feb. 2001.

[7] J. Wang, A. B. Rad, P. T. Chan, "Indirect adaptive fuzzy sliding mode control: Part I: fuzzy switching," *Fuzzy Set and System*, vol. 122, pp. 21-30, 2001.

[8] M.R. Emami, A. A. Goldenberg, and I. B. Turksen, "A robust model-based fuzzy logic controller for robotic manipulators," *Proc. IEEE Int. Conf. Robotics Automation*, vol. 3, 1998, pp. 2500-2505.

[9] S. B. Choi and J. S. Kim, "A fuzzy-sliding mode controller for robust tracking of robotic manipulators," *Mechatronics*, vol. 7, no. 2, pp. 199-216. 1997.

[10] B. W. Bekit, J. F. Whidborne, and L. D. Seneviratne, "Fuzzy sliding mode control for a robot manipulator," Proc. *IEEE Int. Symp. Computational Intelligence Robotics Automation,*, vol. 1, pp. 320-325, 1997.